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Students' metacognitive activities in solving the combinatorics problem: the experience of students with holist-serialist cognitive style

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Abstract. Metacognition is related to improving student learning outcomes. This study describes students' metacognitive activities in solving the combinatorics problem. Two undergraduate students of mathematics education from STKIP PGRI Banjarmasin were selected as the participants of the study, one person has a holist cognitive style and the other a serialist. Data were collected by task-based interviews where the task contains a combinatorial problem. The interviews were conducted twice using equivalent problem at two different times. The study found that the participants showed metacognitive awareness (A), metacognitive evaluation (E), and metacognitive regulation (R) that operated as pathways from one function to another. Both, holist and serialist, have metacognitive activities in different pathway. The path of metacognitive activities of the holist is AERCAE-AAEER-ACRECECC-AREERCE with the AERAE-AER-ARE-ARERE pattern, while the path of metacognitive activities of the serialist is AERCA-AAER-ACRERCERC-AREEEE with the AERA-AER-ARERER-ARE pattern. As an implication of these findings, teachers/lecturers need to pay attention to metacognitive awareness when they begin a stage in mathematical problem solving. Teachers/lecturers need to emphasize to students that in mathematical problem solving, processes and results are equally important.

1 Introduction

Metacognition is used to refer to the awareness individuals have of their own thinking, their evaluation of that thinking, and their regulation of that thinking [1]. Metacognitive awareness (A) relates to individuals' awareness of where they are in the process of solving a problem. It also includes their knowledge of what needs to be done, what has been done, and what might be done in problem solving situations. Metacognitive evaluation (E) refers to judgements about one's thinking processes, capacities and limitations in problem solving situations. For example, individuals could be making a judgement regarding the effectiveness of their thinking or of their strategy choice. Such an evaluative function assumes some awareness of the individuals' thinking processes and anticipates the possible regulation of those processes. Metacognitive regulation (R) draws upon individuals' knowledge about self and strategies, including how and why they use particular strategies, and uses executive skills (such as planning and setting goals) to optimise the use of their own cognitive resources.

Metacognition plays a role in mathematical problem solving. Metacognition is related to improving student learning outcomes [2][4]. Silver argued that failure or success in solving mathematical problems is influenced by the use of metacognition [5]. Similarly, Cardelle-Elawar reports that most students who had difficulties in mathematics less use cognitive or metacognitive strategies [6].



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An experimental study reported that there were differences in mathematics achievement based on students' cognitive style. The study also reported differences in the use of cognitive and metacognitive strategies based on cognitive styles [7]. Another study showed that good critical thinkers engaged in more metacognitive strategies, especially high-level planning and high-level evaluating strategies [8]. Among various dimensions of cognitive styles, Pask's Holist-Serialist is influential to student learning and problem solving [9]. Therefore, this study aims to describe students' metacognitive activities in solving a combinatorics problem based on Holist-Serialist cognitive style. According to Biryukov, combinatorial problems have non-algorithmic characters. Solving problems of this domain is useful for developing mathematical reasoning, critical thinking abilities, and thus it leads to activating their metacognitive skills [10].

2. Method

This is an explorative research with qualitative approach. The qualitative approach was chosen because the study was natural, no treatment was given to the subjects. In addition, the data are descriptive in the form of a series of words and pictures taken from interviews, field notes, photos, videos, and documents.

Two undergraduate students of mathematics education from STKIP PGRI Banjarmasin were selected as the participants of the study, one person has a holist cognitive style and the other a serialist. Selection of the participants using a questionnaire adapted from the Study Preference Questionnaire (SPQ) developed by Ford as revised by Clarke [11].

Data were collected by task-based interviews where the task contained a combinatorial problem. The problem solving process is divided into 4 stages from Polya, that is understanding the problem, devising a plan, carrying out the plan, and looking back [12]. At each stage, subjects are interviewed to reveal their metacognitive activities during this stage.

The interviews were conducted twice using equivalent problem at two different times. Data from interviews supported by observations, answer sheets, and recordings (audio and video) were analyzed qualitatively, including data review, categorization, reduction, display and interpretation.

The participants were given a task containing a combinatorial problem, number 1 for the first interview and number 2 for the second.

1. How many different 4-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if digits 8 and 9 always appear in every number and repetitions are not allowed?
2. How many different 5-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if digits 2 and 5 always appear in every number and repetitions are not allowed?

The results showed that the participants' responses in both interviews were consistent. Therefore, the following will only be exposed to the results of the first interview.

3. Result and discussion

3.1. Metacognitive activities of the holist (H)

3.1.1. Understanding the problem. H was assigned a task of combinatorial problem. To understand the problem, H said that first he will analyze the problem by reading it. This means H showed metacognitive awareness (A), i.e. he knows what to do to understand the problem. Furthermore, H said that he would read the problem with quick and silent. When asked why using that strategy, H reasoned that he would concentrate on finding important information in the matter. He also believes that by quick reading he would be faster and easier to understand the problem. That way is also his habit. This information indicates that he judged the effectiveness of the strategy he will use in reading to understand the problem (E) before he decided to use the strategy (R).

After he tried to find the information (C = cognition) by reading the given problem, H explained his understanding of the given problem. He showed in which part of the problem he obtained the

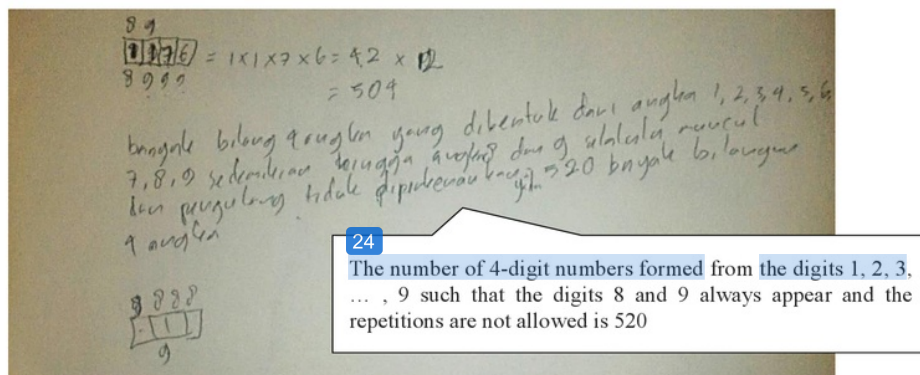
information (by pointing). He explained the meaning of 'repetitions are not allowed', "If there are 4 boxes, the digits in the boxes should not be the same." His responses indicate that he already has knowledge of what is known and asked, even though it is not mentioned explicit. Thus, it can be said that he knows about the problem (A). Furthermore, he judged the correctness of his understanding of the problem (E) by expressing his belief that his understanding is correct. This is reinforced by an explanation of his understanding as mentioned earlier.

It appears that the pathway of metacognitive activities of subject H is AERCAE. If cognitive activity C is ignored, then the pattern of metacognitive activities of H in understanding the problem is AERAE. This pattern not only complements what Wilson and Clarke have founds, AER, AERE and ARE in overall problem-solving activities [1], but also explains that 10 metacognitive activities that includes awareness, evaluation, and regulation has arisen in the process of understanding the problem.

3.1.2. Devising a plan. After he understood the problem, H stated that he was thinking about how to do it, imagining work processes, and the end result. This means that he knows what to do (A) before solving the given problem. He claimed that he knows some of ways in which he can solve the problem (A), but the way he described as "making boxes" was the first way he thought and he was not sure to work using the other way. This shows that H judged the effectiveness of the way he would use to solve the problem (E). He also expressed his confidence to apply the way. Again, this indicates that he has judged, not only on the effectiveness of the way to be used, but also himself capacity (E). Furthermore, based on the judgments, H sets out to use the "make boxes" way as he would use to solve the given problem (R).

The path of metacognitive activities of H in devising a plan is AAEEER with an AER pattern (ignoring the same sequential repetition of activities). This pattern is a standard pattern proposed by Wilson and Clarke [1]. He said that metacognitive evaluation is often preceded by metacognitive awareness and anticipating metacognitive regulation. However, these results show that these three metacognitive activities all appear in this stage, which in the study by Wilson and Clarke are still not clearly visible.

3.1.3. Carrying out the plan. After he setted out his plan to solve the problem, H said that he would start working according to the plan. It showed that H knows what to do (A). Next, H started to solve the problem (C), and he worked on his plan (R).



Handwritten solution by subject H:

8 9
 $\boxed{1}\boxed{1}\boxed{7}\boxed{6} = 1 \times 1 \times 7 \times 6 = 42 \times 12$
 $8999 = 504$

bagian bilang 9 angka yang dibentuk dari angka 1, 2, 3, 4, 5, 6
 7, 8, 9 sedemikian sehingga angka 8 dan 9 selalu muncul
 dan pengulangan tidak diperbolehkan $\Rightarrow 20$ banyak bilangannya
 9 angka

3 8 8 8
 $\boxed{1}\boxed{1}\boxed{1}\boxed{1}$
 9

24
 The number of 4-digit numbers formed from the digits 1, 2, 3, ... , 9 such that the digits 8 and 9 always appear and the repetitions are not allowed is 520

Figure 1. Solution by H

H realized that there was a mistake by said, "... I repeat, I realize there is a mistake here, it should be 8, 9 written above. Here 1, 1, since 8, 9 each represent a single digit." This indicates that he judged the correctness of his thought process (E). The H's statement indicates that the boxes he created should be written with 'the number of digits that can be placed in a particular position', not even with

‘digits to be placed.’ Therefore, he corrected it (C) by writing the digit 8 above the first box and 9 above the second box, and then replacing the contents of the first and second boxes with 1 (Figure 1). H continued his work (C), but he again realized there was a mistake, “... in here there are three possibilities, in here there are three possibilities. So there are... 3, 6, 9. It should be here multiplied by 9 (counting while mumbling). Oh ... it should be here 12.” This indicates an evaluation by H on his mistaken thought process (E) followed by correcting the error (C) by replacing 42×9 to 42×12 (Figure 1). Then H continued the work until it is done (C). In solved the problem, he has used the principle of multiplication.

It appears that the pathway of metacognitive activities of H is ACRECCECC with ARE pattern. This pattern is in line with Wilson and Clarke's finding [1]. This finding also show that metacognitive awareness, metacognitive evaluation, and metacognitive regulation, all three can appear at this stage in combinatorial problem solving.

3.1.4. Looking back. After he solved the problem, H said, “Now I want to check.” It showed that he knows what to do (A). Then he planned the check (R). Next, he checked his work (E). This can be seen in his response, “I pay attention to my work from the beginning, how I do it, the process is like that, right. The calculation (he checked the calculation), is correct.” This quote also shows that H judged the correctness of his thought process in solving the problem (E) indicated by the words “right” and “correct”, supported by his statement, “I think this answer is correct.”

To make sure, then he thought of different ways (R) to find answers. After he found the answer in a different way (C), then he compared the result with the previous answer. This shows that he judged the correctness of his thought process in solving the problem (E) by comparing the results of the two solutions he made, even though the latter solution was not written. Based on the checks, he believed that the solution is correct.

It appears that the pathway of metacognitive activities of H is AREERCE with ARERE pattern. This pattern not only complements what Wilson and Clarke have founds, AER, AERE and ARE in overall problem-solving activities [1], but also explains that metacognitive activities that includes awareness, evaluation, and regulation has arisen in the process of looking back.

3.2. Metacognitive activities of serialist (S)

3.2.1. Understanding the problem. S was assigned a task of combinatorial problem. To understand the problem, he said that first he would read the problem. This means that he showed metacognitive awareness (A), i.e. he knows what to do to understand the problem. S said that he would read silently from beginning to end. When asked why he should read like that, he reasoned that it was his habit and made it easy for him to understand the problem. This information indicates that S judged the effectiveness of the way he will used in reading to understand the problem (E) before he decided to use the way (R).

After he tried to find the information (C) by reading the given problem, 127 explained his understanding of the problem. He said, “From this problem, I know ... so there are 4-digits numbers that can be formed from digits 1 through 9, and digits 8 and 9 should always appear in the formation of these numbers. So, whether it appears at the beginning or at the end, they should appear, there are always digits 8 and 9.” According to S, the digits appear only once, because on the question there is the phrase “repetitions are not allowed.” This quote indicates that S understands the problem (A).

It appears that the pathway of metacognitive activities of S is AERCA with AERA pattern. This pattern not only complements what Wilson and Clarke have founds [1], but also explains the metacognitive activities that includes awareness, evaluation, and regulation has arisen in the process of understanding the problem.

3.2.2. Devising a plan. After he understood the problem, S stated that he tried to find the right formula to do the problem, thought to solve the problem, and devised steps to work. This means that S knows

what to do (A) before solving the problem. Furthermore, he recalled materials related to the problem, similar problems, also previous and successful work strategies (A). Before decided to use "make boxes" way, he believed that his strategy can be used to solve the problem. That is, S judged the effectiveness of the strategy he will used (E), and then decided to use it (R), the same way he used to solve similar problems.

The path of metacognitive activities of S in devising a plan is AAER with an AER pattern. This pattern is a standard pattern proposed by Wilson and Clarke [1]. He said that metacognitive evaluation is often preceded by metacognitive awareness and anticipating metacognitive regulation.

3.2.3. Carrying out the plan. After he setted out the plan, S said he would carry out his plan. It shows that he knows what to do (A). Next, he started to solve the problem (C), and he worked on the plan (R).

S changed his strategy from 'writing all possibilities' to 'writing only some parts.' Furthermore, he utilized the patterns he found to calculate the many possible 4-digit numbers if the numbers 8 and 9 in different positions (Figure 2). He said that the reason changed the strategy, "because I cannot write it down anyway, because after I know 'oh it is 6 times', so I immediately write, oh in here 6, definitely" and "I shortened by writting down the codes." This indicates that S judged the effectiveness of the strategy used to solve the problem (E), and then for that reason S changed its strategy (R).

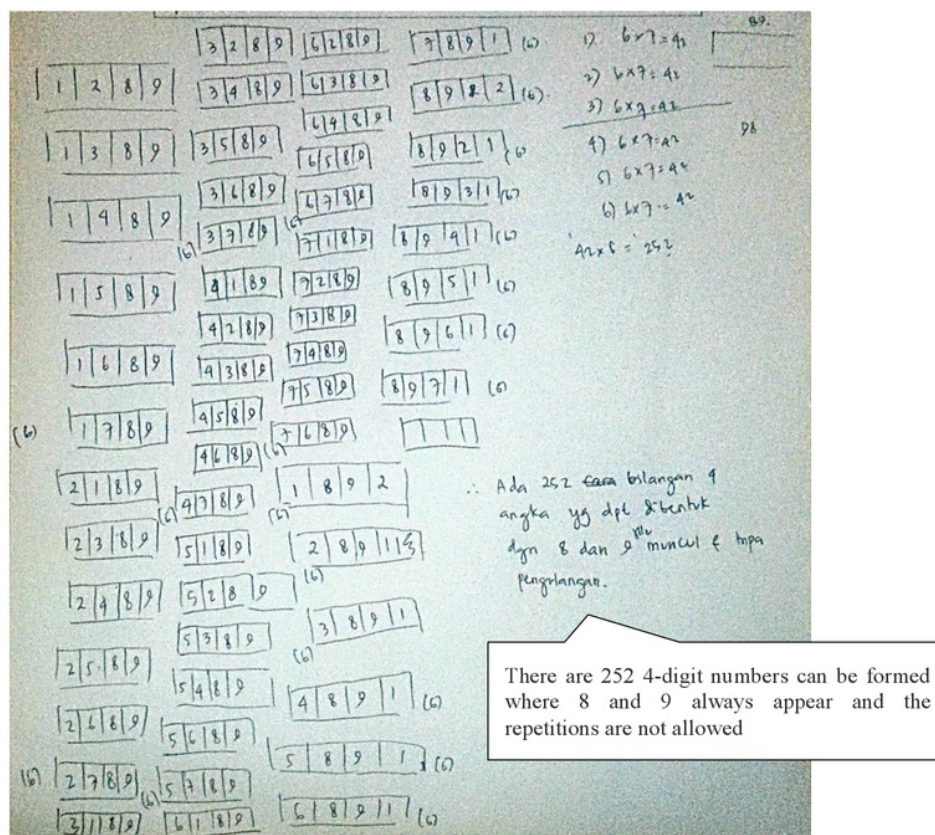


Figure 2. Solution by S

Furthermore, S worked with a new strategy (C) until he finds a pattern similarity if positions 8 and 9 are exchanged. He said, "... Then when the position is reversed, it means there are also the same digit pairs." He realized that his work will be shorter (Figure 2). The response of the S indicates that he, once again, judged the effectiveness of his strategy and realized that there was a new strategy that makes his work shorter (E), so he changed his strategy again (R). Then he worked on a new strategy (C) until finally he found the answer, 252.

It appears that the pathway of metacognitive activities of H is ACRERCERC with ARERER pattern. This pattern complements Wilson and Clarke's findings [1]. This finding also shows that metacognitive awareness, metacognitive evaluation, and metacognitive regulation, all three can appear at this stage in combinatorial problem solving.

3.2.4. Looking back. After he solved the problem, S claimed to need to check. It shows that he knows what to do (A). Then, he planned the check (R), "... I looked back at the list I made." Next, he checked his work (E).

S believed that the answer is correct, because he has made a list of possibilities in sequence, "I do not think there is any more because at the time of doing this I do it in sequence." S also said, "After I finished checking and I am sure with the answer then I make the conclusion." It can be interpreted that S judged the truth of his thought process in solving the problem (E) and he believed the correctness of the answer (E).

It appears that the pathway of metacognitive activities of H is AREEEE with ARE pattern. This pattern is in line with Wilson and Clarke's finding [1] and also explains that metacognitive activities that include awareness, evaluation, and regulation have arisen in the process of looking back.

Both participants, holist and serialist, showed metacognitive activities in different pathways. The path of metacognitive activities of the holist is AERCAE-AAEER-ACRECCCECC-AREERCE with the AERAE-AER-ARE-ARERE pattern, while the path for the serialist is AERCA-AAER-ACRERCERC-AREEEE with the AERA-AER-ARERER-ARE pattern. Wilson & Clarke [1] and Barbacena & Norina [13] did not elaborate the problem-solving process in the stages, so the results clarify their findings by detailing the paths and patterns at each stage of problem solving.

In each stage, metacognitive activities begin with a metacognitive awareness. This indicates that metacognitive awareness is a prerequisite for other metacognitive activities, i.e. regulation and evaluation. As an implication, teachers/lecturers need to pay attention to metacognitive awareness when they begin to understand the problem, devising a plan, carrying out the plan, and looking back.

Both show more metacognitive evaluation activities than metacognitive awareness and regulation. This is in line with the findings of Barbacena and Norina [13]. This can be explained as the tendency of participants to result rather than process, whereas in solving a mathematical problem, results and processes must receive balanced attention. Further, both subjects use different strategies to solve the problem, especially at the stages carrying out the plan and looking back.

4. Conclusions and recommendations

In solving the combinatoric problem, holist and serialist showed metacognitive awareness, metacognitive evaluation, and metacognitive regulation that operated as pathways from one function to another. Both, holist and serialist, have metacognitive activities in different pathways. However, in every stage it always begins with metacognitive awareness activities. As an implication, teachers/lecturers need to pay attention to metacognitive awareness when they begin a stage in mathematical problem solving. They show more metacognitive evaluation activities than metacognitive awareness and regulation. This can be explained as the tendency of participants to result rather than process, whereas in solving a mathematical problem, result and process must receive balanced attention. Therefore, teachers/lecturers need to emphasize to students that in mathematical problem solving, processes and results are equally important.

Mathematical problem solving involves the complex interaction between cognitive and metacognitive activities. While this study is more focused on metacognitive activities. Therefore, in the future, studies can be developed to know the interaction, for example by observing the interaction between thinking skills (cognition) and metacognitive function, so more can be utilized to improve student learning outcomes.

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